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Oscillations of three neutrinos with all $\Delta m^2 \sim 10^{-3}\,\mathrm{eV}^2$

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Abstract: Oscillations of three neutrinos with all squared mass splittings around $10^{-3}\,\mathrm{eV}^2$ are not firmly excluded by solar neutrino experiments. We carefully verify that they are also perfectly compatible with atmospheric neutrino experiments: due to accidental reasons the SuperKamiokande experiment is rather insensitive to 'solar' ν_e/ν_μ oscillations, even if some characteristic small effects could become visible with more statistics. This pattern of oscillations can be excluded by new solar experiments, or cleanly discovered at KamLand. We also perform a fit of the most recent atmospheric SK data within the usual assumption that 'solar' effects are negligible.

Keywords: Beyond Standard Model, Neutrino Physics, CP violation.

Contents

1. Introduction

In this paper we explore the possibility that the atmospheric and solar neutrino deficits can be produced by oscillations of the three known neutrinos with comparable mass splittings $\Delta m^2 \sim 10^{-3}\,\mathrm{eV}^2$ and large ν_e/ν_μ and ν_μ/ν_τ mixings. In section 2 we recall why solar neutrino experiments do not exclude this possibility. In section 5 we show that this possibility is also perfectly consistent with atmospheric neutrino experiments, even if it is sometimes said that, since SuperKamiokande (SK) sees no anomaly in the rate of ν_e events, significant oscillations of atmospheric ν_e neutrinos are excluded. This is not the case [1]. In short the reason is the following: atmospheric neutrinos are produced by cosmic rays in the following proportion

$$(N_{\nu_e}, N_{\nu_\mu}, N_{\nu_\tau}) \propto (1, R, 0)\,.$$

Since $R \approx 2$ the nearly maximal ν_μ/ν_τ oscillation responsible of the atmospheric ν anomaly gives oscillated neutrinos with composition $\propto (1,1,1)$ — the only proportion not affected by further possible oscillations. Of course this argument is only approximate (R is larger than 2 for $E_\nu \gtrsim 1\,\mathrm{GeV}$ and the ν_μ/ν_τ oscillation need not be exactly maximal): a numerical computation will confirm that the conclusion is correct.

From a theoretical point of view, the possibility of explaining neutrino anomalies with comparable Δm^2 has important consequences. It is easy to build models that naturally explain large mixings, but between neutrinos with comparable mass. It is also easy to build models that give hierarchical neutrinos, but with small mixings. It is more difficult to obtain large mixing angles ($\theta_{23} \sim 1$) between hierarchical neutrinos ($\Delta m^2_{23} \gg \Delta m^2_{12}$, all oscillation parameters are precisely defined later on in eq. (4.1)): only few mass matrices (justifiable with various symmetries) naturally give this pattern [1, 2]. Thus, non hierarchical neutrinos would not give very restrictive indications on flavour physics.

The paper is structured as follows. In section 2 we explain why a large solar $\Delta m^2 \sim 10^{-3}\,\mathrm{eV}^2$ is not safely excluded by solar neutrino experiments. In section 3 we describe how we will fit the SK data. Since this is a delicate task, in section 4 we test our procedure performing a complete fit of the most recent SK data, in the 'standard' case where the solar Δm^2_{12} is negligibly small. In section 5 we show that SK data are perfectly compatible with a Δm^2_{12} as large as allowed by the Chooz bound [3], and we discuss how its effects can be detected at SK and at future experiments.

2. Energy independent solar oscillations?

If solar and atmospheric neutrino experiments are not affected by unknown systematic errors or by rare statistical fluctuations, if the standard solar models (SSMs) are correct, if there are only the three known light neutrinos, then forthcoming neutrino experiments will confirm and measure more precisely that

$$\begin{array}{ll} \Delta m^2_{23} \approx 10^{-3}\,\mathrm{eV}^2 \qquad & \sin^2 2\theta_{23} \approx 1 \\ \Delta m^2_{12} \approx 10^{-10}\,\mathrm{eV}^2 \qquad & \sin^2 2\theta_{12} \approx 1 \end{array}$$

These values give a good fit of atmospheric neutrino data and give an acceptable fit (10% C.L. [5, 6, 7]) of solar neutrino data.

Since these conclusions are quite strong, it is useful to discuss if they are also strongly founded.

The well known 'MSW solutions' with $\Delta m^2 \approx 10^{-5}\,\mathrm{eV}^2$ give a poor fit of the distortion of the solar ^{8}B spectrum observed by SuperKamiokande [6, 7] and are ruled out at 95% C.L. [6]. Maybe this experimental result (or the estimation of its uncertainties) is wrong. Maybe the distortion is not produced by ν oscillations, but is due to a flux of 'hep' neutrinos ~ 15 times higher than what predicted by SSMs [1, 8].

At the moment, it seems more safe to believe that there are at least three (instead of one) possible oscillation solutions to the solar neutrino problem. We now discuss why even this sentence is not strongly founded: it is not safely excluded that the solar neutrino anomaly can be explained by an energy independent $\nu_e \to \nu_e$ survival probability $P_{ee} \sim 1/2$ (as can be produced by a large $\Delta m^2_{12} \sim 10^{-3}\,\mathrm{eV}^2$).

The deficit $r_i = \Phi_i^{\rm exp}/\Phi_i^{\rm BP98}$ of solar neutrinos measured by the three kind of solar experiments (i = Cl, Ga and SK), with respect to the central values $\Phi_i^{\rm BP98}$ of fluxes predicted by the BP98 [9] SSM, are

$$\begin{aligned} r_{\rm Cl} &= 0.315 \pm 0.025 \quad [10] \\ r_{\rm SK} &= 0.47 \pm 0.02 \quad\;\; [11] \\ r_{\rm Ga} &= 0.58 \pm 0.05 \quad\;\; [12] \end{aligned} \tag{2.1}$$

(the errors do not include the SSM uncertainty). The predictions of a solar-model-independent analysis, in presence of an energy-independent P_{ee}, are

$$\begin{aligned} r_{\rm Cl} &= P_{ee}(0.03 + 0.73 R_{^8{\rm B}} + 0.24 R_{^7{\rm Be}}) \\ r_{\rm SK} &= (0.15 + 0.85 P_{ee}) R_{^8{\rm B}} \\ r_{\rm Ga} &= P_{ee}(0.60 + 0.10 R_{^7{\rm Be}} + 0.31 R_{^8{\rm B}}) \end{aligned} \tag{2.2}$$

where $R_\alpha \equiv \Phi_\alpha/\Phi_\alpha^{\rm BP98}$ is the ratio of total flux of type α neutrinos (α = pp, pep, ^{7}Be, ^{13}N, ^{15}O, ^{17}F, ^{8}B, hep) emitted by the sun, with respect to the central value of the BP98 [9] SSM. Only two free parameters $R_{^8{\rm B}}$ and $R_{^7{\rm Be}}$ appear in eqs. (2.2); the others have been eliminated using the fact that the total luminosity of the sun is known, and other solid informations (see [13, 1] for more details).

From eqs. (2.1) and (2.2) we can easily see that

- If BP98 is correct (it predicts $R_{^8{\rm B}} = 1 \pm 0.2$ and $R_{^7{\rm Be}} = 1 \pm 0.05$ [9]), the three experimental data are compatible with an energy-independent P_{ee} only in presence of a very unprobable statistical fluctuation ($p \approx 0.2\%$) [14, 1];

- Even if SSMs are not correct (i.e. $R_{^8{\rm B}}$ and $R_{^7{\rm Be}}$ are treated as free parameters of order one), the experimental data are incompatible with an energy-independent P_{ee} [14, 1]. A decent fit is possible only if SSMs are so wrong ($R_{^7{\rm Be}}$ close to zero) that the solar neutrino problem disappears [1] — a possibility strongly disfavoured by recent helio-sysmological tests of SSMs.

- Solar data can be explained by an energy-independent P_{ee} if BP98 is correct, but one of the experiments is affected by some unknown systematic error [1]. For example, *it is sufficient to double the error quoted by the chlorine experiment [10] to have $P_{ee} = 1/2$ compatible with experimental results* (in presence of a reasonably probable, $p \sim 10\%$, statistical fluctuation).

In conclusion we believe that the evidence for a deficit of solar neutrinos is strong, while there is yet no strong evidence for an energy dependent oscillation of solar neutrinos.

3. Fitting SK data

Reproducing what SK has really measured is a complex and delicate task. We briefly describe how we do the computation. Five basic ingredients are necessary for a fit of the SK data.

1. The experimental data: we use the most recent ones (736 days of data taking) [15].

2. The prediction for the flux of atmospheric neutrinos produced by cosmic rays (see [16]). We include effects due to the magnetic field of the earth and to variation of solar activity.

3. The oscillation probability for neutrinos across the earth and the atmosphere. In our case we have a generic neutrino 3×3 mass matrix with 3 comparable Δm^2: since we know no simple analytic approximation that takes into account all potentially relevant matter effects (the MSW [17] effect and resonances that can affect the neutrinos that cross the mantle and the core of the earth [18]) we include all matter effects with a fully numerical computation. The disadvantage is that the transition probabilities for oscillations with long pathlength $L \gg E_\nu/\Delta m^2$ are rapidly oscillating functions of the neutrino energy. Even for the simplest realistic model of earth density, a numerical averaging (using a sufficiently large number of E_ν-bins) is more efficient than analytic averaging.

4. The cross section and detection efficiencies in the SK detector. A simple and safe technique has been used in [19] for fitting the old Kamiokande data. They employed the energy spectra of parent neutrinos given by the Monte Carlo simulation of the Kamiokande detector. The corresponding data for the much larger SK detector have not been published (without these information it is not possible to know what SK is really measuring), although they are now available at the www address [20]. We take into account that SK measures the neutrino direction with an error that depends on the ν energy [21] ($\delta\theta$ is around 60° in sub-GeV events and 17° in multi-GeV events).

   We do not include in the fit data about 'upward through going muons' [22] because they are subject to larger theoretical uncertainties and they are too energetic for being strongly affected by oscillations (they are however very interesting for excluding alternative explanations of the atmospheric ν deficit [23]).

5. A χ^2 function. This is a delicate point, since it requires an estimation of theoretical uncertainties in neutrino fluxes and correlation between them. For simplicity we stick to the accurate definition of [24] and we will discuss when appropriate the effect of different definitions.

Moreover we impose the CHOOZ bound about disappearance of reactor $\bar{\nu}_e$ [3]. The CHOOZ collaboration measures the annihilation energy $E = E_{\bar{\nu}_e} + m_{\rm p} - m_{\rm n} + (2-1)m_e$ of positrons produced by inverse beta decay $\bar{\nu}_e{\rm p} \to \bar{e}{\rm n}$ for various energy bins between 1 and 7 MeV. Since we will be interested in an oscillation pattern where the $\bar{\nu}_e$ survival probability depends on $E_{\bar{\nu}_e}$, we carefully treat the CHOOZ data, grouping them into two $E_{\bar{\nu}_e}$ bins. We correctly reproduce the 'initial' CHOOZ bound ($\Delta m^2_{12} < 0.9\ 10^{-3}\,{\rm eV}^2$ for maximal mixing at 90% C.L.). A 'final' analysis of the whole CHOOZ data has not yet been presented: with increased statistics, the CHOOZ bound could be improved up to $\approx 0.6\ 10^{-3}\,{\rm eV}^2$ [25].

4. Standard fit of SK data

We now begin to study the SK data assuming, as usual, that Δm^2_{12} is too small to have relevant effects. We start with a standard fit because, beyond being interesting, allows to check our computation with other ones [15, 24, 26]. We find a very satisfactory agreement with [24], where many plots of oscillations effects are presented.

The oscillation parameters are precisely defined in the following way. The neutrino mixing matrix V is parametrized as

$$V = R_{23}(\theta_{23}) \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi} & 0 \\ 0 & 0 & 1 \end{pmatrix} R_{13}(\theta_{13}) R_{12}(\theta_{12})\,, \tag{4.1}$$

where $R_{ij}(\theta_{ij})$ represents a rotation by θ_{ij} in the ij plane and $i = \{1, 2, 3\}$ are three neutrino mass eigenstates of mass m_i. With this parameterization $\theta_{23} \sim 45^\circ$ gives the ν_μ/ν_τ mixing tested at SK; while θ_{1i} produce the deficit of solar ν_e neutrinos (θ_{13} could be zero). We also define $\Delta m^2_{ij} \equiv m^2_j - m^2_i$. We can assume that $|\Delta m^2_{23}|$ is the largest splitting (see [1] for more details). With this parameterization atmospheric oscillations depend only on Δm^2_{23}, θ_{23} and θ_{13}.

In fig. 1 the continuous lines show how the χ^2 depends on Δm^2_{23} (for maximal $\theta_{23} = 45^\circ$ and zero θ_{13}) fitting separately the sub-GeV and the multi-GeV events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to Δm^2_{23} and allows slightly smaller values of Δm^2_{23}. This small difference is probably due to the fact that our fit does not include data about 'upward through going muons' [22]: since our χ^2 has a particularly flat minimum these less significant data, that seem to prefer higher values of $\Delta m^2_{23} \sim 10^{-(3\div 1)}\,{\rm eV}^2$, can cause the small difference.

The dotted lines in fig. 1 correspond to a simplified definition of the χ^2: we fit separately the angular dependence of each one of the four kind of events measured by SK (e-like and μ-like, sub-GeV and multi-GeV), treating the overall normalization of each one of them as free but including only statistical errors. We see that, at least in

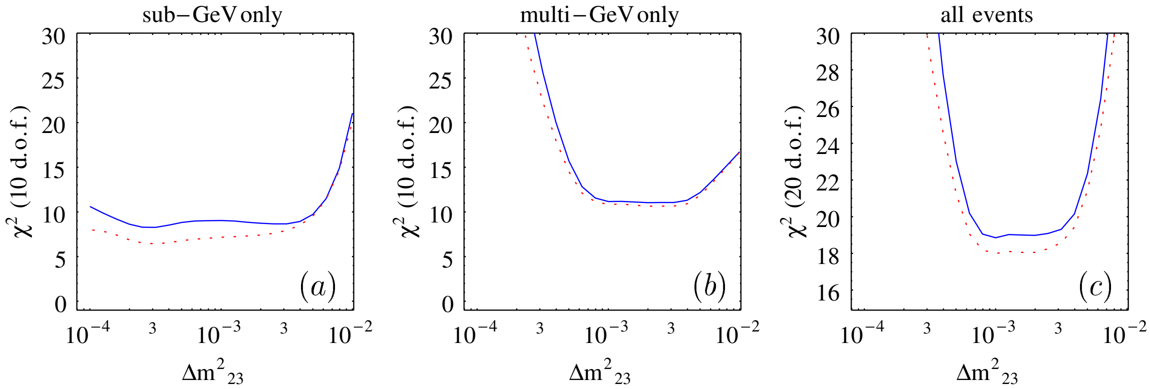

Figure 1: χ^2 of SuperKamiokande data as function of Δm^2_{23} for $\theta_{23} = 45°$, $\theta_{13} = 0$ and negligible Δm^2_{12}. In (a) (resp. (b)) we show the separate contribution from sub-GeV (resp. multi-GeV) events; while all events are used in (c). Continuous and dotted lines correspond to two different definitions of the χ^2 (see the text).

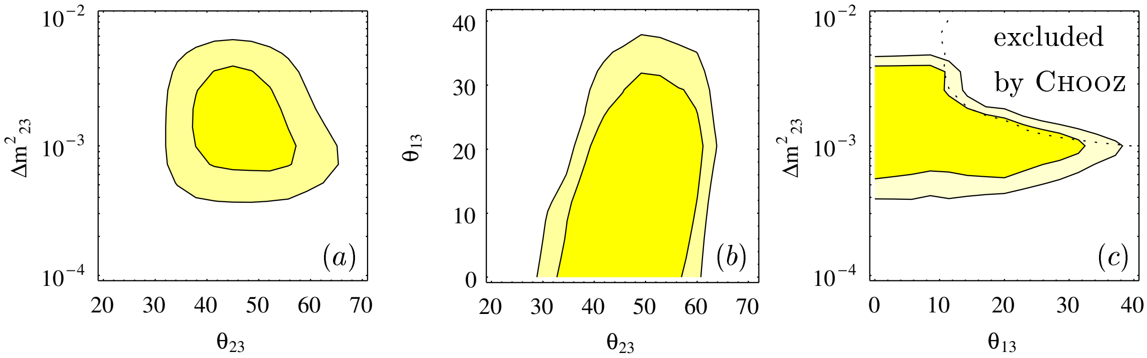

Figure 2: Fit of SK and Chooz data under the assumption that 'solar' oscillations are negligible, so that the only relevant oscillations parameters are Δm^2_{23}, θ_{23} and θ_{13}. Inside the darker (lighter) areas $\chi^2 < 24$ (30) (the χ^2 uses 22 experimental data; the best fit has $\chi^2_{\rm min} = 18$; In (a) the χ^2 is minimized with respect to θ_{13}, in (b) with respect to Δm^2_{23}, and in (c) with respect to θ_{23}.

the fit in fig. 1, there is no significant difference between this simplified χ^2 and the one in [24]. An interesting feature shown by fig. 1 is that *small values of* $\Delta m^2_{23} \lesssim 10^{-3}\,\mathrm{eV}^2$ *(that are difficult to test at planned 'long-baseline' experiments) are now disfavoured by the clean multi-GeV data only* (whose uncertainty is dominated by statistics), rather than by the sub-GeV data (whose interpretation strongly depends on the details of the SK experiment and on theoretical predictions about the neutrino flux). Thus the lower bound on Δm^2 is solid and can be improved with more statistics in the next years.

In fig. 2 we show the results of a fit in the relevant oscillations parameters, Δm^2_{23}, θ_{23} and θ_{13}. The best fit has $\chi^2_{\rm min} = 18$ (the χ^2 uses 20 experimental data from SK and 2 from Chooz). We show contour lines corresponding to the values $\chi^2 = 24$ and $\chi^2 = 30$. Using standard approximations, values of $\chi^2 - \chi^2_{\rm min}$ can be converted into confidence levels that delimit 'best fit regions', and values of χ^2 can be converted into confidence levels that delimit 'exclusion regions'. Shaded areas roughly correspond to $(90 \div 99)\%$ confidence levels[1]. In fig. 2a we minimize the χ^2 with respect to θ_{13} and determine the allowed regions in the plane $(\Delta m^2_{23}, \theta_{23})$. We see that $\sin^2 2\theta_{23} > 0.8$ at 90% C.L. Fig. 2b shows the χ^2 minimized with respect to Δm^2_{23} and determines the allowed values of the mixing angles. In fig. 2c we minimize the χ^2 with respect to θ_{23} and show how the upper bound on θ_{13} depends on Δm^2_{23}. If $\Delta m^2_{23} = 3\ 10^{-3}\,\mathrm{eV}^2$ the CHOOZ bound (dashed lines) requires a small value of $\theta_{13} \lesssim 10°$. For a mass splitting below the CHOOZ bound, $\Delta m^2_{23} \lesssim 10^{-3}\,\mathrm{eV}^2$, a larger θ_{13} is not forbidden by the CHOOZ data, but disfavoured by SK because it generates an up/down asymmetry in the e-like (sub-GeV and multi-GeV) sample. However, as explained in the introduction, the sub-GeV asymmetry vanishes for $\theta_{23} = 45°$, while the one in the multi-GeV sample vanishes for an appropriate value of $\theta_{23} \lesssim 45°$. Since our analysis does not say that $\Delta m^2_{23} = 1 \cdot 10^{-3}\,\mathrm{eV}^2$ is disfavoured, for an appropriate value of $\theta_{23} \approx 45°$, 'large' $\theta_{13} \gtrsim 20°$ are allowed.

5. Non-standard fit of SK data

Finally we assume that the 'solar' Δm^2_{12} is sufficiently large to affect atmospheric neutrinos[2]. The SK observables now depend on all oscillation parameters. For simplicity we only exhibit fits where $\theta_{12} = 45°$ (as suggested by the deficit of solar neutrinos, if θ_{13} is small) and $\theta_{13} = 0$, so that the CP-violating phase ϕ becomes irrelevant. In figs. 3 we show the result of our fit of SK atmospheric data in Δm^2_{13}, Δm^2_{12} and θ_{23}. Fig. 3a is done for $\Delta m^2_{12} = 0.8\ 10^{-3}\,\mathrm{eV}^2$, just below the CHOOZ bound, and shows that a good fit is possible for appropriate values of Δm^2_{13} and θ_{23}. In fig. 3b (resp. c) we minimize the χ^2 with respect to Δm^2_{13} (resp. θ_{23}). The main result is that *values of Δm^2_{12} as large as $10^{-3}\,\mathrm{eV}^2$ are compatible with the most recent SK data.* Infact the upper bound on $\Delta m^2_{12} \lesssim 0.9\ 10^{-3}\,\mathrm{eV}^2$ — shown in figs. 3b,c — comes from the CHOOZ experiment. Even for the maximal value of Δm^2_{12}, 'solar' oscillations have small effect on SK observables: they produce an up/down asymmetry of e-like events (this observable has a dominant statistical error) accompanied by a change in the overall number of events (this observable has a dominant theoretical error). Depending on how the χ^2 is defined this small effect can slightly improve or deteriorate the fit.

[1] We do not insist on the precise correspondence since it has no particular meaning. There is no objective way of converting χ^2 levels into statements like "oscillation parameters lie in the shaded region with 90% probability".

[2] Related analyses, motivated by the fact that the large angle MSW solution allows $\Delta m^2_{12} < +0.2\ 10^{-3}\,\mathrm{eV}^2$ (the positive sign is the one that gives the desired MSW effect for solar neutrinos) have been recently performed in [27, 28]. For such values of Δm^2_{12} our results agree with [28] and disagree with [27].

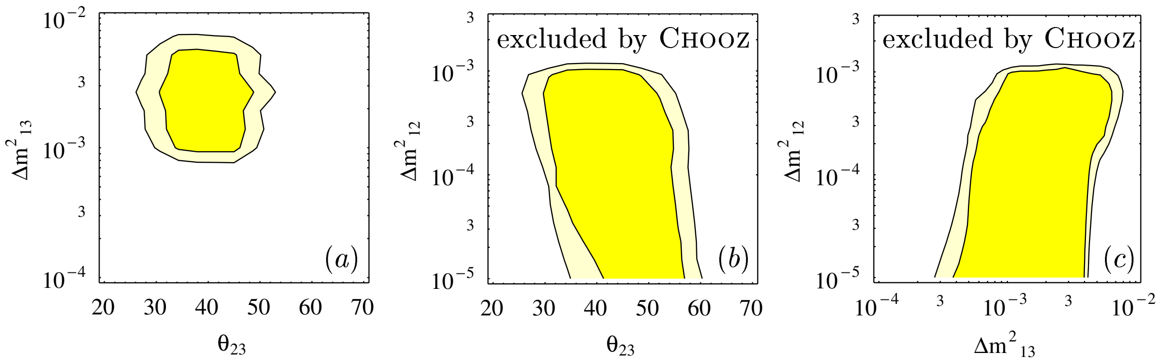

Figure 3: Fit of SK data assuming that all Δm^2 are large enough to affect atmospheric neutrinos. As in fig. 2 we show contour lines of the $\chi^2(\Delta m^2_{ij}, \theta_{ij})$ at $\chi^2 = 24$ and 30 ($\min \chi^2 = 17$). All fits are restricted to $\theta_{13} = 0$ and $\theta_{12} = 45°$. In (a) we show $\chi^2(\Delta m^2_{13}, \theta_{23})$ for a large value of $\Delta m^2_{12} = 0.8\ 10^{-3}\,\mathrm{eV}^2$; in (b) (resp. (c)) we show the χ^2 minimized with respect to Δm^2_{13} (resp. θ_{23}).

These small effects on e-like events produce the preference for values of $\theta_{23} \lesssim 50°$ when Δm^2_{23} is larger, apparent from figs. 3a,b. We now discuss this up/down asymmetry of e-like events in more detail because it is the most promising signal of 'solar' oscillations that can be observed at SK.

The qualitative features of the asymmetry are well reproduced by the rough approximation (again obtained making the simplifying assumptions used in [1])

$$\frac{N_e^\uparrow}{N_e^\downarrow} \approx 1 + \frac{R(1+\cos 2\theta_{23}) - 2}{4} \sin^2 2\theta_{12}. \tag{5.1}$$

As already explained in the introduction, if $R = 2$ (as in the sub-GeV data) and $\theta_{23} = 45°$ there is no effect. Multi-GeV neutrino events have $R \approx 3$; but are rarer and too energetic for being strongly affected by a Δm^2_{12} below the CHOOZ bound. Since the cancellation makes the situation intricate, a numerical computation is necessary to determine the possible signatures of a solar Δm^2 at SK. We show in figure 4 some example of how the quantities measured by SK are affected by 'solar' oscillations for different values of Δm^2_{12} if $\theta_{23} = 45°$. The arrows on the horizontal axes denote the direction of the incoming neutrinos and correspond to the five bins of $\cos\vartheta_{\rm zenith}$ used by the SK collaboration to present their results (for example $\uparrow$ refers to the up-going neutrinos that cross the core of the earth). The rate of each one of the 20 bins is normalized with respect to the no oscillation case (our predictions for unoscillated rates are in satisfactory agreement with the Monte Carlo of the SK collaboration). We see that the effect is very small for any value of Δm^2_{12}.

In figure 5 we again show the effects of 'solar' oscillations, but for the smallest value of $\theta_{23} = 30°$ compatible with the SK data. There are now larger effects in the e sub-GeV sample. We see that a small $\sin^2 2\theta_{23} = 3/4$ gives a poorer fit of μ events; but this fit also depends on the value of Δm^2_{23}: for this reason we do not consider

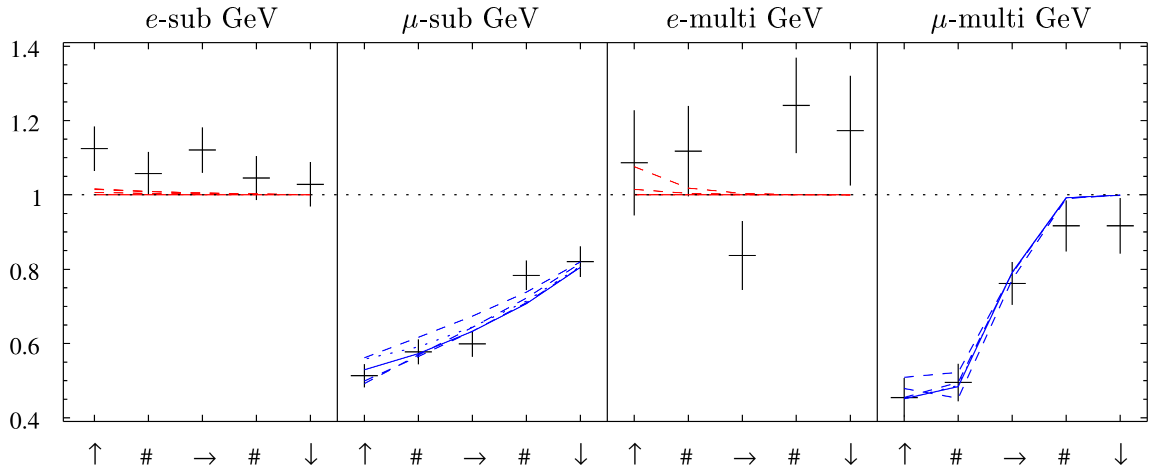

Figure 4: Effect of 'solar' oscillations on SK observables, normalized to the unoscillated rates, for $\theta_{12} = \theta_{23} = 45°$, $\theta_{13} = 0$, $\Delta m^2_{13} = 3\,10^{-3}\,\mathrm{eV}^2$ and $\Delta m^2_{12} = \{0, 0.3, 1, 3, 9\}10^{-4}\,\mathrm{eV}^2$ (larger values have longer dashing). The arrows on the horizontal axes denote the direction of incoming neutrinos. The crosses are the experimental data (their error bars only include statistical errors).

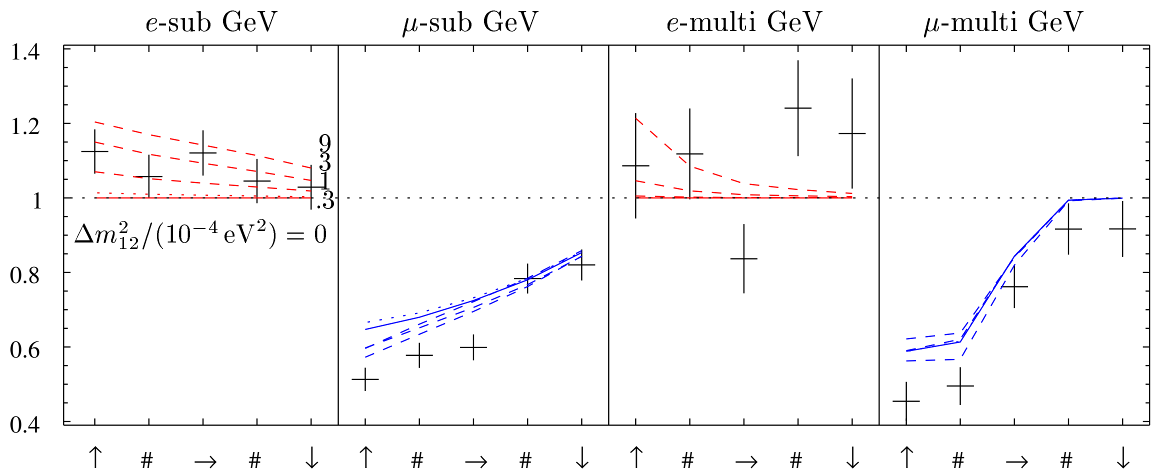

Figure 5: As in fig. 4, but for a smaller $\theta_{23} = 30°$. The results for $\theta_{23} = 60°$ are similar, but the sign of the effects is reversed.

useful discussing small 'solar' effects in the μ events. To correctly interpret fig. 5, we must remind that the overall normalization of the fluxes (i.e. the '1' line in the plot) has a $\sim 20\%$ theoretical uncertainty. Moreover the '1' lines for the four different data samples can be moved independently by $\sim 5\%$. Our χ^2 knows that these systematic uncertainties are highly correlated and says that (with present statistics) even the 'solar' effects shown in fig. 5 are "small effects". For $\theta_{23} = 60°$ the effects due to 'solar' oscillations have similar size, but opposite sign.

In all these computation we have assumed that $\Delta m^2_{23}, \Delta m^2_{12} > 0$. Matter effects do not significantly affect the results if the Δm^2 have different signs.

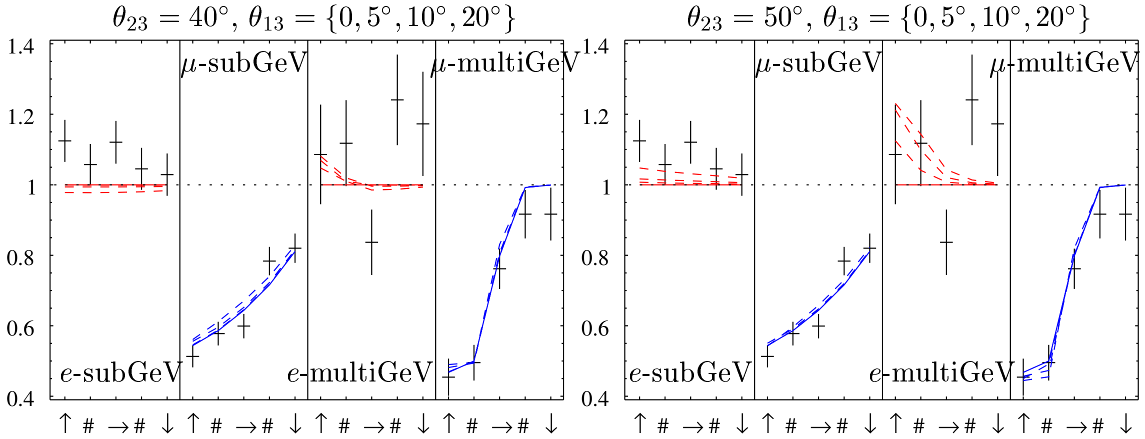

Figure 6: Sample of how the SK observables are affected by a non zero θ_{13} for $\Delta m^2_{23} = 3\ 10^{-3}\,\mathrm{eV}^2$ and $\theta_{23} = 40°$ (left) $\theta_{23} = 50°$ (right). The arrows on the horizontal axes denote the direction of the incoming neutrinos.

5.1 Non zero θ_{13}

So far we have assumed that $\theta_{13}, \phi = 0$. Like in the 'standard' scenario, a small $\theta_{13} \lesssim 20°$ is allowed by the Chooz and SK data. CP violation cannot affect SK observables since integration over neutrino energy averages it to zero.

We now discuss how effects due to θ_{13} mixing can be distinguished from effects due to a 'solar' oscillation. In fig. 6 we show how a $\theta_{13} = \{0, 5°, 10°, 20°\}$ affects the SK observables if $\Delta m^2_{12} \ll \Delta m^2_{23} = 3 10^{-3}\,\mathrm{eV}^2$, $\theta_{23} = \{40°, 50°\}$. Comparing the results of the numerical computation shown in fig. 6 with figs. 4 and 5, we notice two main differences

- θ_{13} oscillations mainly affect multi-GeV e-like events, while 'solar' oscillations can only affect e sub-GeV events.

- For a given value of θ_{23}, 'solar oscillations' and 'θ_{13}-oscillations' produce up/down asymmetries of opposite sign. The up/down asymmetry produced by θ_{13} is different from zero and positive even if $\theta_{23} = 45°$.

(In particular, a not too large θ_{13} can only produce a few % excess of sub-GeV e-like events). Both these features can be understood comparing the approximate up/down asymmetries produced by a small θ_{13},

$$\frac{N_e^\uparrow}{N_e^\downarrow} \approx 1 + 2\theta_{13}^2 (R \sin^2\theta_{23} - 1)\,, \tag{5.2}$$

with the corresponding approximation for 'solar' effects, eq. (5.1). We remind that $\Delta m^2_{23} \gtrsim \Delta m^2_{12}$ and that sub-GeV events have $R \approx 2$, while multi-GeV ones have $R \approx 3$. In both cases the effects cancel out if $R \approx 2$ and $\theta_{23} \approx 45°$.

6. Conclusions

In conclusion a large 'solar' $\Delta m^2_{12} \lesssim 10^{-3}\,\mathrm{eV}^2$ is not safely excluded by solar neutrino experiments and is allowed by atmospheric neutrino experiments.[3] A 'solar' oscillation has little effect on SK observables and can slightly ameliorate or deteriorate the fit of SK data, depending on how the χ^2 is defined. Its most clear signature at SK is an up to $\sim 15\%$ angular dependent excess (or deficit) of e sub-GeV events. An indication for a $\sim 10\%$ excess of e-like events in the sub-GeV sample was present in the first year of data taking at SK; but the evidence has decreased in the most recent analyses with doubled statistics. However this happened in a not very nice way:

- The SK collaboration has introduced small improvements in their Monte carlo, and obtained slightly different predictions;
- The rate of e sub-GeV events in the most recent part of the sample (last 321 days of data taking) is $(18\pm5)\%$ lower than in the first part (first 414 days) [29].

Moreover, if the overall flux of atmospheric ν is somewhat lower than what current estimates indicate — as suggested by a recent (preliminary) measurement of cosmic ray fluxes [30] — than SK is observing a smaller deficit of ν_μ and an excess of ν_e events. In conclusion we believe that the normalization of ν fluxes is still very uncertain and that an excess (or even a deficit) of e sub-GeV events could be present. With more statistics it will be possible to use the unoscillated down-going multi-GeV events to fix the normalization, or to search for up/down asymmetries in the e samples.

To conclude, a large 'solar' Δm^2_{12} can be experimentally investigated in different ways:

- **SuperKamiokande**, when high-statistics will be available, could see some indication in the details of e events;
- future **solar experiments** (like Borexino) could exclude this possibility;
- **KamLand** could soon observe an evident deficit of reactor $\bar{\nu}_e$;
- '**long-baseline experiments**' (like K2K and Minos) can confirm the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation seen at SK. These experiments could also see the ν_e appearance signal due to a sufficiently large θ_{13}. On the contrary ν_e appearance[4] due only

[3] On the contrary no acceptable fit of SK data is possible for the mass pattern $\Delta m^2_{12} \sim 10^{-3}\,\mathrm{eV}^2$ and $\Delta m^2_{23} \sim \mathrm{eV}^2$, sometimes invoked for reconciling the unconfirmed LSND oscillation with solar and atmospheric ones. We do not show any numerical result because this fact is sufficiently clear from the approximate analysis in [1].

[4] The ν_μ disappearence observed at SK reduces the ν_e appearance signals at K2K and Minos by a factor close to 2. This explains why, according to fig 7b, these experiments are less sensitive to $\nu_\mu \to \nu_e$ oscillations than in standard estimates, done using the 'two-neutrino approximation' where ν_τ are neglected.

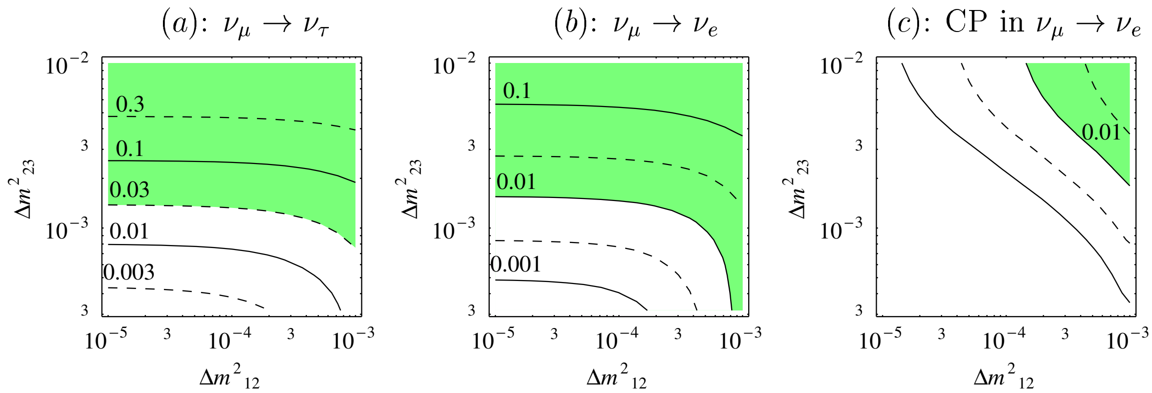

Figure 7: (a)Contour plots of the transition probabilities $P(\nu_\mu \to \nu_\tau)$, (b) CP-conserving part of $P(\nu_\mu \to \nu_e)$ and (c) CP-violating part of $P(\nu_\mu \to \nu_e)$, at MINOS ('medium' beam) as function of the mass splittings $(\Delta m^2_{12}, \Delta m^2_{23})$ for maximal CP-violatng phase $\phi = 90°$, maximal $\theta_{12} = \theta_{23} = \pi/4$ and moderately large $\theta_{13} = 20°$. For this value of θ_{13}, $\Delta m^2_{23} \gtrsim 3\ 10^{-3}$ is disfavoured by CHOOZ. MINOS can explore the shaded regions.

to 'solar' effects can be seen only if Δm^2_{12} is very close to its maximal value, $10^{-3}\,\mathrm{eV}^2$. CP violating effects in $\nu_\mu \to \nu_e$ are too small to affect the experiment, except maybe in some extreme case [31]. All these conclusions are illustrated by fig. 7, where we show contour-plots of the relevant transition probabilities averaged over the energy spectrum of the 'medium energy' ν_μ beam at MINOS (see the caption for more details).

- future '**neutrino factories**' (beam of $\bar{\nu}_\mu \nu_e$ produced by decay of muons) [32], if Δm^2_{12} is large enough, could study CP violation and make precision measurements of the oscillation parameters (but it could be difficult to distinguish θ_{13} effects from Δm^2_{12} effects).

Note added. In the recent paper [33] it was observed that if $\Delta m^2_{23} \approx 10^{-3}\,\mathrm{eV}^2$ the (interesting?) scenario named 'tri-maximal mixing' (in our language $\theta_{12} = \theta_{23} = \pi/4$ and $\sin^2\theta_{13} = 1/3$, i.e. $\theta_{13} \approx 35°$) is still compatible with SK and CHOOZ data (at least until the complete CHOOZ data will be presented), due to certain cancellations. This is also shown by fig. $2c$: as we have discussed, these cancellations are not a special feature of tri-maximal mixing. Quite generally SK is rather insensitive to new oscillations beyond the observed one.

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